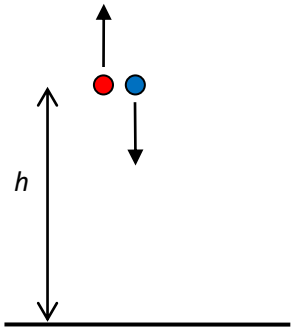
Problem of the week
Kinematics

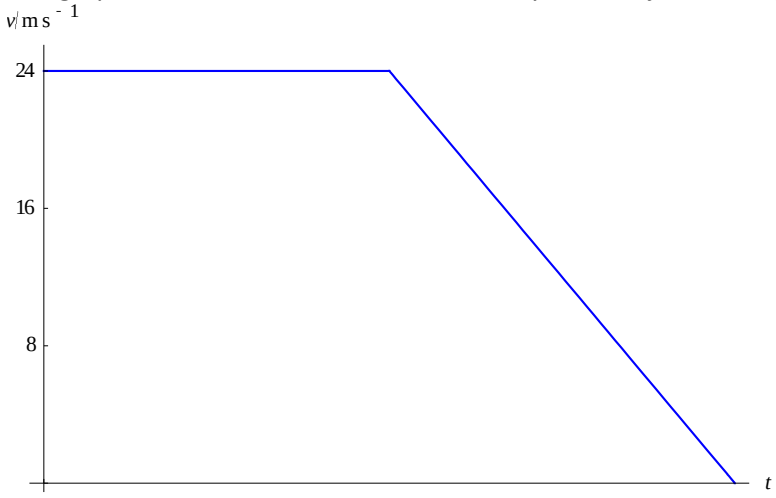
For all parts except the last take $g = 10\ \text{m s}^{-2}$.

(a) Two balls are thrown with the same initial speed of 30 m s^{-1} from the same height *h* above ground. One ball is thrown upwards and the other downwards.



The red ball reaches the ground *T* seconds after the blue ball. Determine *T*.

(b) The graph shows the variation of the velocity of an object moving on a straight line.



(i) The distance covered is 108 m. The deceleration is 8.0 m s^{-2}. Determine the duration of the motion.

(ii) Plot a graph to show the variation of position with time. Take the initial position to be zero.

(c) A ball is dropped from a large height above ground. Air resistance is not negligible. The ball reaches terminal speed. Sketch a graph that shows the variation with time of the position of the ball.

(d) A ball is thrown upwards from the edge of a cliff with speed 30 m s^{-1}. The ball reaches the sea below the cliff after 8 s. Air resistance is negligible.

(i) Determine the height of the cliff.

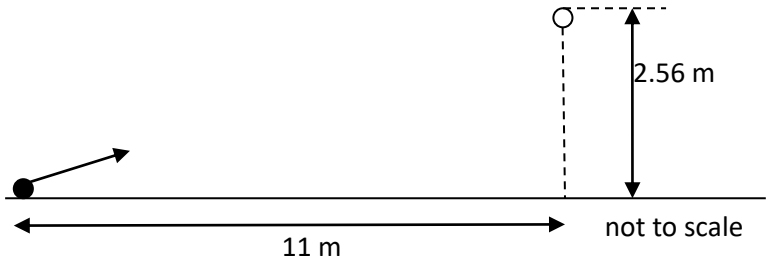
(ii) Calculate the speed of the ball as it hits the sea.

(iii) Estimate the average speed of the ball.

(e) A projectile is launched vertically upwards. At times t = 2.0 s and 4.0 s the height of the projectile above ground is the same. Determine the maximum height of this projectile.

(f) A projectile is launched horizontally with speed 20 m s^{-1}. Determine the angle between the velocity and the horizontal after 2 s.

(g) The top of the horizontal crossbar in football is 2.56 m from the ground. A penalty kick is taken at 11 m from the middle of the goal line. The ball is kicked with speed 25 m s^{-1} at an angle of 19° to the horizontal. The ball travels along a vertical plane through the middle of the goalpost. The radius of a soccer ball is 12 cm.



Determine if there will be a goal.

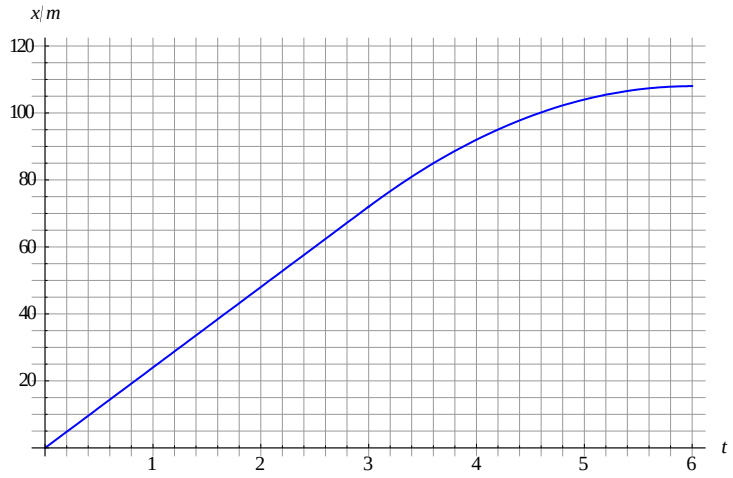
Answers

(a) The extra time T is the time it takes the blue ball to reach its maximum height and return to its launch point. It takes $0 = 30 - 10t \Rightarrow t = 3.0$ s to get to the top so T = 6.0 s.

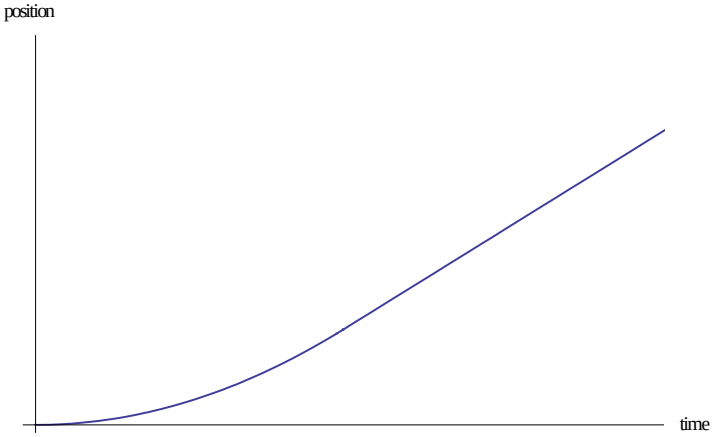
(b)

(i) Let τ be the time at which deceleration starts and T the required time for the entire motion. The deceleration is $\frac{0-24}{T-\tau} = -8.0$, hence $T - \tau = 3.0$. Then $108 = \frac{1}{2}(T+\tau)\times 24 \Rightarrow T + \tau = 9.0$. Hence, $T = 6.0$ s.

(ii) Straight line until t = 3 s, followed by concave down parabola with zero slope at t = 6 s.



(c)



(d)

(i) From $y = y_0 + ut + \frac{1}{2}at^2$ we get $y = 0 + 30\times 8 - \frac{1}{2}\times 10\times 8.0^2 = -80$ m. The height is 80 m.

(ii) $v = u + at = 30 - 10 \times 8.0 = -50 \text{ m s}^{-1}$. The speed is 50 m s^{-1}.

(iii) The height above the cliff is $0 = 30^2 - 2 \times 10 \times y \Rightarrow y = \frac{900}{20} = 45 \text{ m}$. The distance travelled is 45+45+80 = 170 m so the average speed is $\frac{170}{8.0} \approx 21 \text{ m s}^{-1}$.

(e) Let the initial velocity be *u*. Then at *t* =2 s the velocity is $v_2 = u - 10 \times 2.0$ and at *t* = 4 s it is $v_4 = u - 10 \times 4.0$. We have that $v_2 = -v_4$ so $u - 10 \times 2.0 = -(u - 10 \times 4.0)$ or $2u = 60$ so $u = 30 \text{ m s}^{-1}$. Then $0 = 30^2 - 2 \times 10 \times y \Rightarrow y = \frac{900}{20} = 45 \text{ m}$.

(f) The vertical velocity component is $v_y = 0 - 10 \times 2.0 = -20 \text{ m s}^{-1}$. The angle with the horizontal is $\tan\theta = \left|\frac{v_y}{v_x}\right| = \left|\frac{-20}{20}\right| = 1 \Rightarrow \theta = 45^\circ$.

(g) Time of travel is $\frac{11}{25\cos 19^\circ} = 0.4654 \text{ s}$. Vertical position of center of ball at this time is $y = 0.12 + 25\sin 19^\circ \times 0.4654 - \frac{1}{2} \times 9.8 \times 0.4654^2 = 2.85 \text{ m}$. Bottom of ball is at a height of $2.85 - 0.12 = 2.73 \text{ m}$ so the ball goes over the crossbar. Something like Roberto Baggio's heartbreaking (for some of us) last penalty kick in the 1994 World Cup final.